DEVELOPING ALGORITHMS IN FUZZY ENVIRONMENT

A

PROJECT REPORT

in the faculty of Mathematical Sciences

Under CPE Scheme

Submitted by

Namarta

Assistant Professor

P.G. Department of Mathematics

G.S.S.D.G.S Khalsa College, Patiala

Declaration

I hereby affirm that the work "**Developing algorithms in fuzzy environment**"presented in this project is exclusively my ownand there are no collaborators. It does not contain any work for which a degree/diploma has been awarded by any other university/Institution. All the ideas and references have been duly acknowledged.

Date :

Namarta

Assistant Professor

P.G. Department of Mathematics

KhalsaCollege , Patiala.

ACKNOWLEDGEMENT

This project would not have been possible without the support of many people. I would like to thank all of them. I would like to express my gratitude to our worthy principal **Dr. Dharminder Singh Ubha** for his motivation and advice to work on a project. I extend my sincere thanks to **Dr. JasleenKaur**and **Dr. Ravinderjit Singh** for their timely and invaluable support during the course of this project work.

I am deeply thankful to **Dr. Umesh Chandra Gupta**, Associate Professor and Head, Department of Mathematics, Shivalik College of Engineering, Dehradun and **Dr. NehaIshesh**, GovtMohindra College Patiala, for their expert and valuable advices on the project work. I am deeply thankful to my colleagues especially **Dr. Gurmeet Singh** and **Dr. ParampreetKaur**for their inspiration and proficient support. My special thanks to **Dr. Harish**, Assistant Professor, Thapar University Patiala for his valuable comments on the research work.I am highly indebted to my husband **Dr. Ravi Singla**, he has been a source of love, encouragement and support. Special thanks to my parents and my loving kids **Liza** and **Sambhav**for compromising their time to this project work. Their support and patience during the process are my motivation to continue and finalize my project.

Above all, I bow my head before God almighty for blessing me with necessary requirements to carry out the work successfully.

NAMARTA

LIST OF PUBLICATION

1. **Application of Game problem in Fuzzy Environment**,

   International journal for research in engineering application & Management , Vol 04, Issue 12 , March 2019, Pp. 228-232

2. **A new alpha-cut approach to solve fuzzy game problem**,

   International Journal of Management, Technology and Engineering , Vol 8, Issue X, October 2018, Pp. 2761-2766

CONTENTS

LIST OF ABBREVIATIONS

FN	Fuzzy numbers
TFN	Triangular fuzzy numbers
TrFN	Trapezoidal fuzzy numbers
PFN	Pentagonal fuzzy numbers
OFN	Octagonal fuzzy numbers
DFN	Decagonal fuzzy numbers

CHAPTER 1

INTRODUCTION AND LITERATUTE SURVEY

1.1 INTRODUCTION:

Game theory is a branch of Mathematical analysis. It is a mathematical tool that deals with decision making in situations of conflict and cooperation between intelligent and rational decision-makers. Game theory owes its origin to Mathematician John von Neumann and Economist Oskar Morgenstren and was firstly published in the book entitled "The Theory of Games and Economic Behavior". Theory of games has played an important role in decision making fields such as defense, economics, political science, management etc. In a game problem each player attempts to take best decision by selecting various strategies from the set of available strategies. There are many types of games such as cooperative and non-cooperative games, sequential game, constant sum, static and dynamic games. In game theory, there are two or more players who take a decision and it affect the outcome of each other. In game theory,the decision makers are called players.The traditional game theory assumes the existence of exact payoffs to solve competitive situations.However in the real life game situations such precise information on the payoffs is not available. Due to lack of information, the players are not able to estimate exactly payoffs in real situations. This lack of certainty may be appropriately modeled by using fuzzy set. In such situations all the payoffs are fuzzy. There are many researches on the solution of game theory whose payoffs are fuzzy.

1.2LITERATURE REVIEW:

Fuzzy set theory given by Zadeh [2] is a very useful technique to solve such game situations where the values of payoffs are approximate or fuzzy. He was the first to propose fuzzy set theory. Bellman and Zadeh [3] elaborated on the concept of decision making in the fuzzy environment. Later on, fuzzy methodologies have been successfully applied in a wide range of real world situations. John Nash [1] proved that a finite game problem always has an equilibrium point at which all players select their best actions, when the opponent's choices are given. In interactive environment, Game theory is proved a prime tool for several decision making processes.

Jain [4] was the first to propose method of ranking fuzzy numbers for decision making in fuzzy situations. Yager [5] used the concept of centroids in the ranking of fuzzy numbers. Dhanalaxmi and Kennedy [23] proposed some ranking methods for octagonal fuzzy numbers.Rajarajeswari and Sudha [20] proposed a new method for ranking of fuzzy numbers by using incentre of centroids. Namarta and Neha[32] proposed ranking of hendecagonal fuzzy numbers by using centroid of centroids. Jatinder and Neha [33] described a ranking method for ordering dodecagonal fuzzy numbers by using incentre of centroids. Selvakumari and Lavanya [22] described an approach for solving fuzzy game problem by using octagonal fuzzy numbers.Kumar et al [19] proposed method that integrates the concept of fuzzy ranking and the minimax principle to get imprecise game value. Jana and Roy [39] choose a ranking method to convert trapezoidal fuzzy numbers into crisp numbers and apply it on fuzzy matrix games.Cevikel and Ahlatcioglu[10] considered two person zero sum game which is based on solution of games with payoffs and goals are fuzzy. Chu and Tsao[7] presented a method to rank fuzzy number by finding the area between the centroid and original points.

Thorani et al. [14] used orthocenter of centroids for ordering of generalized trapezoidal fuzzy numbers.Thirucheran et al. [37] considered a two person zero sum game and used ranking criteria to solve it without converting into crisp problem.Kumar et al.[11] proposed an interactive method that integrates the concept of fuzzy ranking and minimax principle to get an imprecise game value. Selvakumari and Lavanya [27] considered a solution of game theory based on ranking of triangular and trapezoidal fuzzy numbers. Rajarajeswari and Sudha [25] using the concept of rank, mode, divergence and spread for ordering of hexagonal fuzzy numbers.Kamble [38] discussed pentagonal fuzzy numbers by using α-cut approach.Vijay et.al [9] introduced a fuzzy relation approach to solve fuzzy matrix games and deduce to two semi-infinite optimization problems. Bector and Chandra [8] focused on the study of applications of areas related to mathematical programming and matrix games theory. Parvathi and

Malathi[15] introduced STIFN'S based on alpha-beta cuts and its arithemetic operations.Sakawa and Nishizakhi [6] considered a two person zero sum game in which fuzzy multiple payoffs are involved. In their paper maximum-minimum strategy of degree of attainment is discussed and used Sakawa's method to solve linear programming problem.Seikh et.al [16] focused on applications on TIFN's to non-cooperative bi matrix games and defined the equality relationship between two numbers.Mousavi and Rezvani [29] proposed a ranking method of triangular fuzzy numbers which is based on rank, mode, divergence and spread. Babu et.al [17] describes a method based on area , mode, divergence and spread for ordering of fuzzy numbers. In their paper, Centroid of centroids is used to rank the trapezoidal fuzzy numbers and then find the area of this centroid from the original point. Seikh et.al [28] considered a two person in which payoffs are triangular fuzzy numbers. In this paper a method is introduced based on $\alpha -$ cut of TFN and two bi-objective linear programming are introduced.Sharma and Kumar [31] described an algorithm to find value of the game in fuzzy environment and determine the upper and lower bounds of payoffs. In their paper octagonal fuzzy numbers are converted into intervals by using $\alpha -$ cut method.Kumar and Kumaraghuru[30] considered some operations of triangular fuzzy numbers and find a solution of fuzzy game problem.Soniet. al. [40] deals with the study of terms of game and their fuzzification with the help of fuzzy sets.Krishnaveni and Kandasamy [41] proposed a method to solve game problems without converting it into classical version and payoffs are trapezoidal fuzzy numbers Savitha and George [35] discussed two methods for ranking of fuzzy numbers based on values of fuzzy numbers and extent fuzzy analytic hierarchy process.In their paper arithmetic mean operations of fuzzy numbers are used and proposed approach is used in multi criteria decision making process.Lofti et.al [34] considered a new approach magnitude based of distance measure for ordering of fuzzy numbers.Monisha and Sangeetha [36] introduced a concept to solve fuzzy game problems with payoffs are pentagonal fuzzy numbers. In their paper by using ranking method fuzzy numbers are converted into crisp and the solved by usingmaxmin principle.Lee and Yun [21] define extension of Zadeh's principle for pentagonal fuzzy numbers and conclude a triangular fuzzy numbers after the addition and subtraction of numbers. Kamble and Venkatesh [24] make a brief survey on fuzzy numbers and their properties. The equivalence relation between two fuzzy numbers has been investigated with the help of definition given by Puri and Raelscu.Rajarajeswari et.al [18] introduce a new operation on Hexagonal Fuzzy number on the basis of alpha cut sets and operation for addition, subtraction and multiplicationof fuzzy numbers. Cevikel and Ahlatcioglu [12] deals with multiobjective two-person zero-sum games in which payoffs and goals are fuzzy. This paper explains new concepts of solutions for multiobjective two-person zero-sum games with fuzzy payoffs and fuzzy goals. Beaulaand Priyadharsini [26] considered intuitionistic trapezoidal fuzzy numbers based on $(\alpha,\beta)-$cut and also defined operations as addition, multiplication, scalar multiplication, subtraction etc., including exponentiation, extracting nth root, and taking logarithm. Nayak and Pal [13] considered intuitionistic fuzzy programming problems and gives applications of two person matrix games for the solution with mixed strategies. In fuzzy environment ranking of fuzzy numbers is very important aspect of decision making. Many authors have proposed different methods for ranking of fuzzy numbers.

CHAPTER 2

PRELIMINARIES

In this chapter, we will discuss some of the important definitions and terms, which will be used throughout the research.Agameis a situation in which the competitors are in finite numbers. Each player has a finite number of strategies available to him and will determine the outcome of the game. A game takes place when each competitorselect his strategy. In zero sum game, the sum of gains or losses of all the player is zero. If the sum is not equal to zero then it is called non zero sum game.Strategy set of player defines what strategies are available for them to play.

Definitions 2.1

(i) **Pure Strategy:**A pure strategy is a set of rules or options which a player can choosewithin a game, where the outcome is dependent on that option.A pure strategy profile must include one and only one strategy for every competitor. The outcome is also dependent on the actions of the opponent and their choice in the game. A players strategy helps determine what action they are going to play within the game. If a player always chooses the same strategy each time, it is referred to as pure strategy.

(ii) **Mixed strategy:**Mixed strategy is a probability distribution that assigns to each available action a likelihood of being selected.A mixed strategy is the strategy that allows the player to assign probability to each of the pure strategy used, and this strategy is used when the opponent can guess the next move to make benefit from it. In this strategy a player select a combination of two or more strategies.

(iii) **Fuzzy Set:**Let X = {x} denote a collection of objects denoted generically by x. Then a fuzzy set $\widetilde{A}$ in X is a set of ordered pairs $\widetilde{A}$ = {(x, $\mu_{\tilde{A}}$ (x)); x ∈ X} where $\mu_{\tilde{A}}$(x) is termed as the grade of membership of x in A and $\mu_{\tilde{A}}$: X → M is a function from X to a space M which is called membership space. When M contains only two points, 0 and 1, A is non-fuzzy and its membership function becomes identical with the characteristic function of a non-fuzzy set.

(iv) A Fuzzy set $\widetilde{A}$ of universe set X is normal if and only if $Sup_{x \epsilon X}\ \mu_{\tilde{A}}$ (x) = 1.

(v) A fuzzy set $\widetilde{A}$ in universal set X is called convex iff

$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min[\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)]$for all $x_1, x_2 \in$ X and $\lambda \in$ [0,1] .

(vi) A fuzzy set $\widetilde{A}$ of universal set is a fuzzy number iff it is normal and convex.

(vii) A fuzzy number$\widetilde{A}$ = (m, n, α, β)$_{LR}$ is said to be an LR flat fuzzy number if its membership function isgiven by:

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), & x \leq m, \alpha > 0 \\ R\left(\frac{x-n}{\beta}\right), & x \geq n, \beta > 0 \\ 1, & m \leq x \leq n \end{cases}$$

L and Rare called reference functions, which are continuous, non-increasing functions that defining the left and right shapes of$\mu_{\tilde{A}}$(x)respectively and L(0)=R (0) =1.

(viii) **Triangular Fuzzy numbers:** A generalized fuzzy number

$\widetilde{A_T} = (a_1, a_2, a_3 \,; w\,)$ is said to be triangular fuzzy number if its membership function $\mu_{\widetilde{A_T}}$(x) is given below:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x \le a_1 \\ w\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ w & x = a_2 \\ w\left(\frac{a_3-x}{a_3-a_2}\right) & a_2 \le x \le a_3 \\ 0 & x \ge a_3 \end{cases}$$

(ix) Trapezoidal fuzzy number: A generalized fuzzy number $\widetilde{A_{Tr}} = (a_1, a_2, a_3, a_4 \,; w\,)$ is said to be trapezoidal fuzzy number if its membership function $\mu_{\widetilde{A_{Tr}}}$(x) is given below:

$$\mu_{\widetilde{A_{Tr}}} = \begin{cases} 0 & x \le a_1 \\ w\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ w & a_2 \le x \le a_3 \\ w\left(\frac{a_3-x}{a_4-a_3}\right) & a_3 \le x \le a_4 \\ 0 & x \ge a_4 \end{cases}$$

(x) Pentagonal fuzzy number: A generalized fuzzy number $\widetilde{A_P} = (a_1, a_2, a_3, a_4, a_5;\ w\,)$ is said to be pentagonal fuzzy number if its membership function $\mu_{\widetilde{A_p}}$(x) is given below:

$$\mu_{\widetilde{A_P}}(x) = \begin{cases} 0 & x \le a_1 \\ \frac{w}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ \frac{w}{2} + \frac{w}{2}\left(\frac{x-a_2}{a_3-a_2}\right) & a_2 \le x \le a_3 \\ w & x = a_3 \\ w - \frac{w}{2}\left(\frac{x-a_3}{a_4-a_3}\right) & a_3 \le x \le a_4 \\ \frac{w}{2}\left(\frac{a_5-x}{a_5-a_4}\right) & a_4 \le x \le a_5 \\ 0 & x \ge a_5 \end{cases}$$

(xi) Heptagonal Fuzzy Numbers: A generalized fuzzy number $\widetilde{A_H} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7; w)$ is said to be heptagonal fuzzy number if its membership function $\mu_{\widetilde{A_H}}$(x) is given below:

$$\mu_{\widetilde{A_H}}(x) = \left\{ \begin{matrix} 0 & x \le a_1 \\ \frac{w}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ \left(\frac{w}{2}\right) & a_2 \le x \le a_3 \\ \frac{w}{2}+\frac{w}{2}\left(\frac{x-a_3}{a_3-a_2}\right) & a_3 \le x \le a_4 \\ w & x = a_4 \\ \frac{w}{2}+\frac{w}{2}\left(\frac{a_5-x}{a_5-a_4}\right) & a_4 \le x \le a_5 \\ \left(\frac{w}{2}\right) & a_5 \le x \le a_6 \\ \frac{w}{2}\left(\frac{a_7-x}{a_7-a_6}\right) & a_6 \le x \le a_7 \\ 0 & x \ge a_7 \end{matrix} \right\}$$

(xii) Octagonal Fuzzy Numbers:

A generalized fuzzy number $\widetilde{A_O} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7; w)$ is said to be octagonal fuzzy number if itsmembership function $\mu_{\widetilde{A_O}}$(x) is given below:

$$\mu_{\widetilde{A_O}}(x) = \left\{ \begin{matrix} 0 & x \le a_1 \\ \frac{w}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ \left(\frac{w}{2}\right) & a_2 \le x \le a_3 \\ \frac{w}{2}+\frac{w}{2}\left(\frac{x-a_3}{a_4-a_3}\right) & a_3 \le x \le a_4 \\ w & a_4 \le x \le a_5 \\ \frac{w}{2}+\frac{w}{2}\left(\frac{a_5-x}{a_6-a_5}\right) & a_5 \le x \le a_6 \\ \left(\frac{w}{2}\right) & a_6 \le x \le a_7 \\ \frac{w}{2}\left(\frac{a_7-x}{a_8-a_7}\right) & a_7 \le x \le a_8 \\ 0 & x \ge a_8 \end{matrix} \right\}$$

2.2 Operations of Triangular fuzzy numbers:

Let $\widetilde{A_T}$ =(a_1, a_2, a_3) and $\widetilde{B_T}$ =(a_1, a_2, a_3) are two triangular fuzzy numbers. The addition, subtraction, scalar multiplication is defined as under:

Addition:$\widetilde{A_T} + \widetilde{B_T} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

Subtraction:$\widetilde{A_T} - \widetilde{B_T} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$

Scalar Multiplication:$k\ \widetilde{A_T} = (k\ a_1, k\ a_2, k\ a_3)$

2.3 Algorithms to solve game problem:

2.3.1 Maximin-Minimax Principle: There are many approaches to find optimal strategies and value of the game.In two person zero sum game, the principle of maximin-minimax is used to obtain optimal pure strategies and the saddle point. Player A is maximization player and the choice of strategy for player A is called maximin principle. Player A select the strategy that maximizes his minimum gains. In the other hand, Player B is looser player and the choice of strategy for B is called minimax principle. Player B selectthat strategy that minimizes his maximum losses. If the maximin value and minimax value are equal to each other than in this case there exist a saddle point and the optimum pure strategies.

Now maximin and minimax are not always lead to an equilibrium point. In such cases, the player select the optimal mixed strategies to find the value of the game.

Consider an $m \times n$ game with payoff matrix (a_{ij}) whose saddle point does not exist and strategies are mixed Let player A will play his m moves with the probabilities$p_1, p_2, p_3 \dots . p_m$; $p_i \geq 0$, $\sum_{i=1}^{m} p_i = 1$ and is assumed to be the gainer player and player B will play his n moves with the probabilities $q_1, q_2, q_3 \dots . q_n$; $q_j \geq$

0 , $\sum_{j=1}^{n} q_j = 1$ and is assumed to be the looser player. It is assumed that each player has to choose strategy from amongst the pure strategies. The minimum of column maximum and the maximum of row minima does not always lead to the saddle point. Consider the optimal mixture of available strategies to get the equilibrium point. Every player chooses his best strategy to get maximum gains or minimum loses b. The expected payoff to a player with matrix $(_{b_{ij}})$ of $m \times n$ order is defined as:

$$\tilde{E} = \sum_{i=1}^{m} \sum_{j=1}^{n} (p_i b_{ij} q_j)$$

A two person zero sum game of any order matrix without any saddle point can be reduced to 2×2 matrix by using dominance principle. In dominance principle the inferior strategies are dominated by superior one. So a player has no incentive to choose those inferior strategies. In this case the size of the payoff matrix can be reduced by deleting those strategies which are dominated by the others. When the payoff matrix reduces to 2×2 matrix then the probability of optimum strategies and value of the game can be find by the following method:

	Player B		
Player A		B_1	B_2
	A_1	a_{11}	a_{12}
	A_2	a_{21}	a_{22}

The player A has optimum mixed strategies with probability $p_1$and $p_2$and the player B has Optimum mixed strategies with probability q_1 and $q_2$are defined as $p_1 = \frac{a_{22}-a_{21}}{a_{11}+a_{22}-(a_{12}+a_{21})}$, $p_2 = \frac{a_{11}-a_{12}}{a_{11}+a_{22}-(a_{12}+a_{21})}$ *and*

$$q_1 = \frac{a_{22}-a_{12}}{a_{11}+a_{22}-(a_{12}+a_{21})}, q_2 = \frac{a_{11}-a_{21}}{a_{11}+a_{22}-(a_{12}+a_{21})}$$

Where $p_1 + p_2 = 1$ and $q_1 + q_2 = 1$. The value of the game to player A is defined as

V = $\frac{a_{11}a_{22}-a_{12}a_{21}}{a_{11}+a_{22}-(a_{12}+a_{21})}$

2.3.2 Graphical method to solve 2× n and $m \times 2$ Games:

The graphical method is used to find the solution of games whose payoff matrix is of 2× n and $m \times 2$ type matrix Games.this method is applicable only if number of rows or columns are two. This method enables to reduce the size of game into 2× 2 and then solve the problem by any game method. For maximin, Player A should select the probability of strategies to maximize his minimum payoffs. In the similar way player B will select the values probabilities to minimize his expected maximum payoffs. To find the maximinpoint of the lower envelop gives the maximin for player A and the minimum point of the upper envelop gives the minimax for player B. The two lines passing through these points determine the optimum mixed strategies of both the players and the reduced 2× 2 matrix gives the value of the game.

2.3.3 Dominance Property: Dominance property is used to reduce the payoff matrix of any order, it does not affect the optimum strategies of the game. In game, there is always one strategy that is inferior or superior to the other one. It means that player would have no incentive to use inferior strategies as these strategies are dominated by the superior strategies. In such cases, we can reduce the size of payoff matrix by deleting the dominated strategies. Even if a strategy is also inferior to the average of any other strategies then the inferior strategy can be deleted.

2.3.4 Arithmetic (oddment) Method for n × n Games:

When a game matrix cannot be reduced in size by using the principle of dominance, then an easy method to solve game problem is arithmetic method as follows:

Step1. Let A = (a_{ij}) be an n × n payoff matrix. Obtain a new matrix C, whose columns are obtained by subtracting successive columns of matrix A from its proceeding columns.

Step2. Obtain a new matrix R, whose rows are obtained by subtracting successive rows of matrix A from its proceeding rows.

Step3. Determine the oddments corresponding to ith row of A is defined as the determinant $|C|$, where C_i is obtained from C by deleting its ith row.

Similarly, determine oddments$\left|R_j\right|$, where R_j is obtained from R by deleting its jth column.

Step4. Determine the magnitude of oddments corresponding to each row and column of A and write the oddments against their respective rows and columns.

Step5. If the sum of row oddments is equal to sum of column oddments, obtain optimum strategies by the oddments expressed as fractions of the grand total otherwise method fails.

Step6. Calculate the expected value of game corresponding to the optimum strategy.

2.4 Mathematical Formulation of Fuzzy Game Problem:

Consider a two person zero sum fuzzy game in which all the entries in the payoffs matrix are fuzzy numbers. Let player A has 'm' strategies and player B has 'n' strategies. Here it is assumed that each player has to choose strategy from the pure strategies. Player A is always assumed to be gainer and player B is always looser. The payoff matrix m $\times n$ is

$$A = \begin{bmatrix} \tilde{a}_{11}\tilde{a}_{12} & \tilde{a}_{13} & \dots. & \tilde{a}_{1n} \\ \tilde{a}_{21}\tilde{a}_{22} & \tilde{a}_{23} & \dots. & \tilde{a}_{2n} \\ \tilde{a}_{31}\tilde{a}_{32} & \tilde{a}_{33} & \dots. & \tilde{a}_{3n} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.. \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \tilde{a}_{m1}\tilde{a}_{m2} & \tilde{a}_{m3} & \dots. & \tilde{a}_{mn} \end{bmatrix}$$

CHAPTER 3

A NEW APPROACH TO SOLVE FUZZY GAME PROBLEMS AND ITS APPLICATION

3.1 INTRODUCTION

Thischapter deals with computational procedure to rank octagonal fuzzy numbers by using incentre of centroids. Fuzzy game problem in which payoffs are octagonal fuzzy numbers are converted into crisp problem and then solve it by using any traditional game theory method. Chu and Tsao [7] presented a method to rank fuzzy number by finding the area between the centroid and original points Thorani et al. [14] used orthocenter of centroids for ordering of generalized trapezoidal fuzzy numbers. A new method to rank fuzzy numbers by using incentre of centroids is proposed by Rajarajeswari and Sudha[20].Dhanalaxmi and Kennedy [21] described some ranking methods for octagonal fuzzy numbers.Jatinder and Neha[33] presented an ordering of dodecagonal fuzzy numbers with incenterof centroids. Namarta and Neha[32] proposed a method to rank hendecagonal fuzzy numbers by using centroid of centroids. Selvakumari and Lavanya[27] used octagonal fuzzy numbers to solve fuzzy game problem.Thirucheran et al. [37] considered a two person zero sum game and used ranking criteria to solve it without converting into crisp problem. This chaperdescribes the method of ranking octagonal fuzzy numbers using centroid of centroids andincenterof centroids. In octagonal fuzzy number, firstly the octagon is split into two trapezoidal and one hexagon and then computes the centroid of these plane figures. Secondly, it computes the centroid of these centroids and the centroid is followed by calculation of incenter. In this chapter, the method of ranking fuzzy numbers with an area between the incenter and the original point is also introduced.

Thischapter is organized into different sections. Section 3.2 presents octagonal fuzzy numbers and ranking method in which procedure to find incenter of centroids is described. In Section 3.3, applications of ranking of octagonal fuzzy numbers to fuzzy game problems are described.

3.2 PROPOSSED RANKING METHOD:

Octagonal Fuzzy Numbers:A generalized fuzzy number $\widetilde{A_o} = (a_1, a_2, a_3, a_4, a_5, a_6, a_{7,} a_8 ; w)$ is said to be octagonal fuzzy number if its membership function $\mu_{\widetilde{A_o}}$(x) is given below:

$$\mu_{\widetilde{A_o}}(x) = \left\{ \begin{array}{ll} 0 & x \leq a_1 \\ \frac{w}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \leq x \leq a_2 \\ \left(\frac{w}{2}\right) & a_2 \leq x \leq a_3 \\ \frac{w}{2} + \frac{w}{2}\left(\frac{x-a_3}{a_4-a_3}\right) & a_3 \leq x \leq a_4 \\ w & a_4 \leq x \leq a_5 \\ \frac{w}{2} + \frac{w}{2}\left(\frac{a_5-x}{a_6-a_5}\right) & a_5 \leq x \leq a_6 \\ \left(\frac{w}{2}\right) & a_6 \leq x \leq a_7 \\ \frac{w}{2}\left(\frac{a_7-x}{a_8-a_7}\right) & a_7 \leq x \leq a_8 \\ 0 & x \geq a_8 \end{array} \right\}$$

To find the balancing point of the octagon, firstly, divide the octagon into two trapezoidal AIJD, EMNH and one hexagon DJKLME (Fig 1.) and then find the centroid of these plane figures. Let the centroid of these plane figures be G_1, G_2 *and* G_3 respectively. The Centroid of centroids, that is, point G, is taken as the point of reference to define the ranking of generalized octagonal fuzzy numbers. Further the incentre of centroids G_1, G_2 *and* G_3 is also calculated. Then ranking of octagonal fuzzy numbers is defined by using incentre of centroids. Consider the generalized octagonal fuzzy number $\hat{A}_o = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\ ;\ w)$

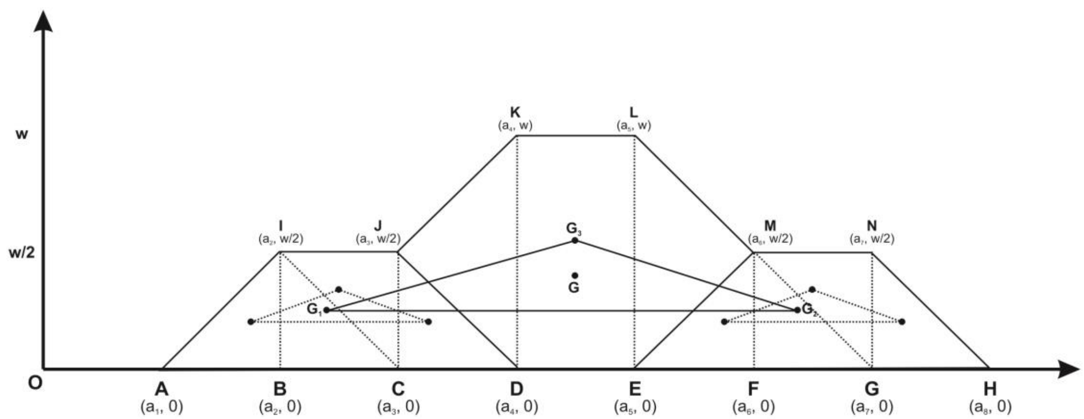


Fig.1

The centroid of these figures are

$$G_1 = \left(\frac{2a_1+7a_2+7a_3+2a_4}{18}, \frac{7w}{36}\right);$$

$$G_2 = \left(\frac{2a_5+7a_6+7a_7+2a_8}{18}, \frac{7w}{36}\right);$$

$$G_3 = \left(\frac{2a_4+a_3+2a_5+a_6}{6}, \frac{w}{2}\right)$$

As G_1, G_2 and G_3 are non collinear and they form a triangle. Therefore Centroid $G_{\widetilde{A_o}}(\overline{x_0}, \overline{y_0})$ of a triangle is

$$G_{\widetilde{A_o}}(\overline{x_0}, \overline{y_0}) = \frac{G_1 + G_2 + G_3}{3}$$

$$G_{\widetilde{A_o}}(\overline{x_0}, \overline{y_0}) = \left(\frac{2a_1+7a_2+10a_3+8a_4+8a_5+10a_6+7a_7+2a_8}{54}, \frac{11w}{54}\right)$$

As a special case, for heptagon fuzzy number $\widetilde{A_H} = (a_1, a_2, a_3, a_4, a_6, a_7, a_8; w)$ i.e., $a_4 = a_5$, the centroid of centroids of a heptagon is given by

$$G_{\widetilde{A_H}}(\overline{x_0}, \overline{y_0}) = \left(\frac{2a_1+7a_2+10a_3+16a_4+10a_6+7a_7+2a_8}{54}, \frac{11w}{54}\right)$$

Now the Incentre $I_{\widetilde{A_o}}(\overline{x_0}, \overline{y_0})$ of a triangle whose vertices are G_1, G_2 and G_3 is given by

$$I_{\widetilde{A_o}}(\overline{x_0}, \overline{y_0}) = \Big(a_{\widehat{A_o}}\left(\frac{2a_1+7a_2+7a_3+2a_4}{18}\right) + b_{\widehat{A_o}}\left(\frac{2a_5+7a_6+7a_7+2a_8}{18}\right) +$$

$$c_{\widehat{A_o}}\left(\frac{2a_4+a_3+2a_5+a_6}{6}\right),\ a_{\widehat{A_o}}\left(\frac{7w}{36}\right) + b_{\widehat{A_o}}\left(\frac{7w}{36}\right) + c_{\widehat{A_o}}\left(\frac{w}{2}\right)\Big)$$

$$a_{\widehat{A_O}} = \sqrt{\left(\frac{2a_4 + a_3 + 2a_5 + a_6}{6} - \frac{2a_5 + 7a_6 + 7a_7 + 2a_8}{18}\right)^2 + \left(\frac{w}{2} - \frac{7w}{36}\right)^2}$$

$$a_{\widehat{A_O}} = \frac{\sqrt{(6a_3+12a_4+8a_5-8a_6+14a_7-4a_8)^2+(11w)^2}}{36},$$

$$b_{\widehat{A_O}} = \sqrt{\left(\frac{2a_4 + a_3 + 2a_5 + a_6}{6} - \frac{2a_1 + 7a_2 + 7a_3 + 2a_4}{18}\right)^2 + \left(\frac{w}{2} - \frac{7w}{36}\right)^2}$$

$$b_{\widehat{A_O}} = \frac{\sqrt{(-4a_1 - 14a_2 - 8a_3 + 8a_4 + 12a_5 + 6a_6)^2 + (11w)^2}}{36}$$

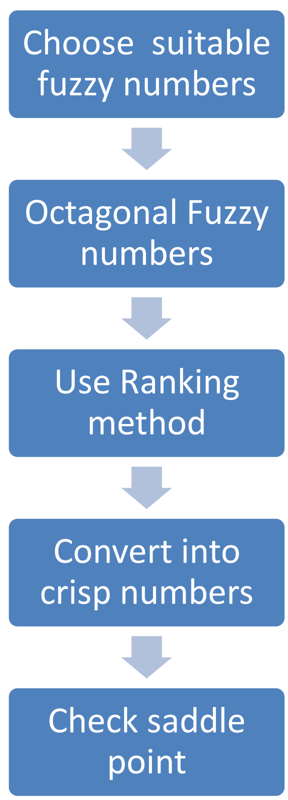
and $c_{\widehat{A_O}} = \sqrt{\left(\frac{-2a_1-7a_2-7a_3-2a_4+2a_5+7a_6+7a_7+2a_8}{18}\right)^2}$

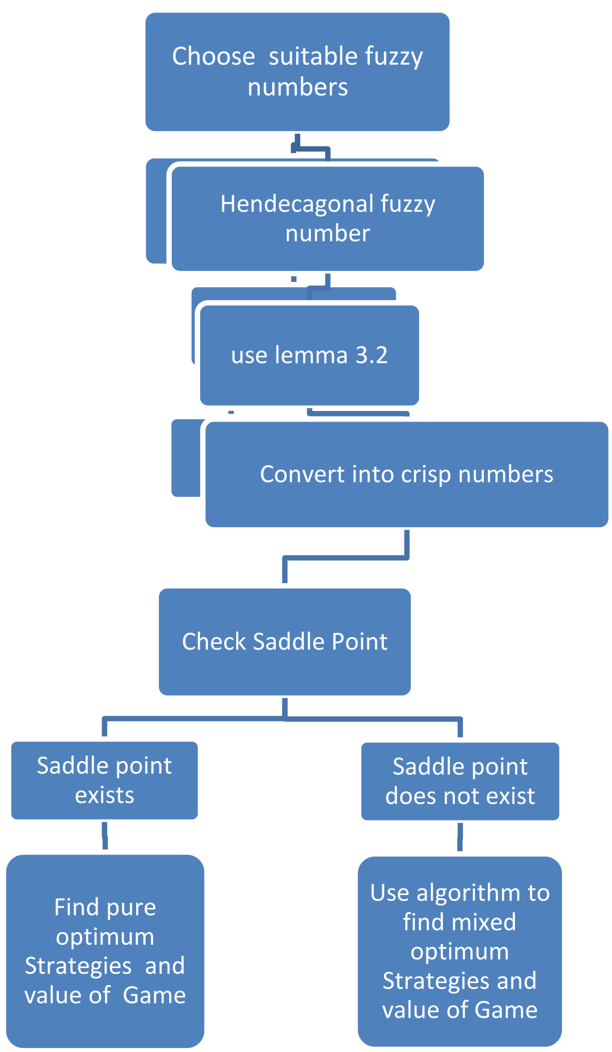
$$c_{\widehat{A_O}} = \frac{-2a_1 - 7a_2 - 7a_3 - 2a_4 + 2a_5 + 7a_6 + 7a_7 + 2a_8}{18}$$

The Ranking function of the generalized octagonal fuzzy number is defined as:

$R\,(\widetilde{A_H}) = \sqrt{\overline{x_0}^2 + \overline{y_0}^2}$

3.3 FLOW CHART FOR SOLVING FUZZY GAME




Choose suitable fuzzy numbers
Hendecagonal fuzzy number
use lemma 3.2
Convert into crisp numbers
Check Saddle Point
Saddle point exists
Saddle point does not exist
Find pure optimum Strategies and value of Game
Use algorithm to find mixed optimum Strategies and value of Game

3.3 Application of ranking of fuzzy numbers to Game theory:

3.3.1 Consider A and B are two players with payoffs octagonal fuzzy numbers:B

$$A\begin{bmatrix}(8,9,10,11,12,13,14,15)(2,3,4,5,6,7,8,9)\\(0,1,2,3,4,5,6,7)(-1,0,1,2,3,4,5,6)\\ \end{bmatrix}$$

By using the ranking method defined in section 3.2, the fuzzy game problem is converted into crisp problem, then solved by maximin-minimax principle and find the value of the game:

$\tilde{a}_{11} = (8,9,10,11,12,13,14,15)$ $\quad$ $M_0^{oct}(\tilde{a}_{11})=$ 11.5

$\tilde{a}_{12} = (2,3,4,5,6,7,8,9)$ $\quad$ $M_0^{oct}(\tilde{a}_{12})=$ 5.5

$\tilde{a}_{21} = (0,1,2,3,4,5,6,7)$ $\quad$ $M_0^{cot}(\tilde{a}_{21})=$ 3.5

$\tilde{a}_{22} = (-1,0,1,2,3,4,5,6)$ $\quad$ $M_0^{oct}(\tilde{a}_{22})=$ 2.5

The reduced payoff matrix is

B

$$A\begin{bmatrix}11.5 & 5.5\\3.5 & 2.5\end{bmatrix}$$

Minimum of 1st row = 5.5

Minimum of 2nd row = 2.5

Maximum of 1st column = 11.5

Maximum of 2nd column = 5.5

Max (min) = 5.5 and Min (max) =5.5

Here Max (min) = Min (max)

It has saddle point. Value of Game = 5.5

3.3.2. Consider A and B are two players with payoffs octagonal fuzzy numbers:

$$\mathrm{B}\,\mathbf{A}\begin{bmatrix} (0,1,2,3,4,5,6,7) & (2,4,5,6,7,8,9,11) & (-1,0,1,2,3,4,5,6) \\ (-3,-1,0,1,2,4,6,7) & (-4,-3,-2,-1,0,1,2,3) & (-3,-2,-1,0,1,2,3,4) \\ (8,9,10,11,12,13,14,15) & (2,3,4,5,6,7,8,9) & (4,5,6,7,8,9,10,11) \end{bmatrix}$$

In section 3.2, the defined ranking method is used convert fuzzy game problem into crisp problem, then solve it by using arithmetic method:

$\tilde{a}_{11} = (0,1,2,3,4,5,6,7)$ $\quad M_0^{oct}(\tilde{a}_{11})=$ 3.5

$\tilde{a}_{12} = (2,4,5,6,7,8,9,11)$ $\quad M_0^{oct}(\tilde{a}_{12})=$ 6.5

$\tilde{a}_{13} = (-1,0,1,2,3,4,5,6)$ $\quad M_0^{oct}(\tilde{a}_{13})=$ 2.5

$\tilde{a}_{21} = (-3,-1,0,1,2,4,6,7)$ $\quad M_0^{oct}(\tilde{a}_{21})=$ 2.2

$\tilde{a}_{22} = (-4,-3,-2,-1,0,1,2,3)$ $\quad M_0^{oct}(\tilde{a}_{22})=$ 0.5

$\tilde{a}_{23} = (-3,-2,-1,0,1,2,3,4)$ $\quad M_0^{oct}(\tilde{a}_{23})=$ 0.5

$\tilde{a}_{31} = (8,9,10,11,12,13,14,15)$ $\quad M_0^{oct}(\tilde{a}_{31})=$ 11.5

$\tilde{a}_{32} = (2,3,4,5,6,7,8,9)$ $\quad M_0^{oct}(\tilde{a}_{32})=$ 5.5

$\tilde{a}_{32} = (4,5,6,7,8,9,10,11)$ $\quad M_0^{oct}(\tilde{a}_{33})=$ 7.5

The payoffs of fuzzy game problem is converted into crisp payoff matrix as:

B

$$\mathrm{A}\quad\begin{bmatrix} 3.5 & 6.5 & 2.5 \\ 2.2 & 0.5 & 0.5 \\ 11.5 & 5.5 & 7.5 \end{bmatrix}$$

Minimum of 1st row = 2.5

Minimum of 2nd row = 0.5

Minimum of 3rd row = 5.5

Maximum of 1st column = 11.5

Maximum of 2nd column = 6.5

Maximum of 3rd column = 7.5

Max (min) = 5.5 and Min (max) =6.5

Here Max (min) $\neq$ Min (max)

It has no saddle point. In order to solve the crisp game problem, we use principle of dominance. As all the elements of first column are greater than third column that means first column is dominated by third column. Hence first column is eliminated, again, all the elements of second row are less than third row, it means third row dominates second row, hence second row is eliminated, we get

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 6.5 & 2.5 \\ 5.5 & 7.5 \end{bmatrix} \end{array}$$

Now 2×2 payoff matrix is obtained. In the reduced matrix, equilibrium point is not obtained, so oddment method is applied to get the value of the game.

Row oddments

6.5 2.5 2

	5.5	7.5	4
Column oddments	5	1	6

Since the sum of column oddments and row oddments is 6. The strategies for both maximization and minimization players are $(\frac{1}{3}, \frac{2}{3})$ and $(\frac{5}{6}, \frac{1}{6})$ respectively and the value of the game is 5.83.

3.4 Conclusion: The chapter describes ranking method for octagonal fuzzy numbers based on area. The process of ranking involves computation of centroids of two trapezoidal figures and one heptagon formed by octagon numbers, then obtaining centroid of these centroids. Finally, the centroid of centroids and incenter of centroids is used to rank octagonal fuzzy numbers for solving fuzzy game problem and illustrated by an example.In the above example 1, the payoffs are octagonal fuzzy numbers. By using the proposed ranking technique these octagonal fuzzy numbers are converted into crisp and then find the saddle point. In this problem, the players A choses his pure strategy A_1 to maximize their minimum gaines and player B choses his B_2 to minimize their maximum losses and the value of the game is 5.5 units. In the second example there does not exist a saddle point, so the player uses their optimum mixed strategies. After converting the payoff matrix into crisp problem, we use dominance property to reduce the payoff matrix into 2×2. Now applying the oddment method and find that Player A will select A_1 and A_3 strategies to maximize his minimum gaines with probabilities 0.33 and 0.67 respectively and player B will select B_2 and B_3 with probabilities 0.83 and 0.17 respectively to minimize his maximum loses. Hence value of the game is 5.83.

CHAPTER 4

A NEW α-CUT APPROACH TO SOLVE FUZZY GAME PROBLEM BY USING PENTAGONAL FUZZY NUMBERS

4.1 INTRODUCTION

The chapter considers a two person zero sum game in which fuzzy payoffs are pentagonal fuzzy numbers. A ranking method is proposed to convert pentagonal fuzzy numbers into crisp number and it is used to solve fuzzy game problems. The fuzzy game problem is converted into crisp problem and then solved by using traditional method. Lee and Yun [21] define extension of zadeh's principle for pentagonal fuzzy numbers and conclude a triangular fuzzy numbers after the addition and subtraction of numbers. Kamble[38] discussed pentagonal fuzzy numbers by using α-cut approach.

The chapter describes the method of ranking pentagonal fuzzy numbers by using α-cut technique. In this chapter, the method of ranking fuzzy numbers is applied on fuzzy game problems. By using this technique fuzzy game problem is converted into crisp problem and then solved by arithmetic method.The paper is organized into different sections. In Section 4.2, some basic definitions of fuzzy set and pentagonal fuzzy numbers are given. Section 4.3 presents proposed ranking method of fuzzy numbers. In Section 4, mathematical formulation of fuzzy game problem is described. Section 5, describes an Arithmetic method to solve game problem. In Section 6, proposed method is illustrated with the help of numerical examples.

4.2 Pentagonal Fuzzy Numbers:

4.2.1. A generalized fuzzy number $\widetilde{A_p}$ =($a_1, a_2, a_3, a_4, a_5;\ w$) is said to be pentagonal fuzzy number if its membership function $\mu_{\widetilde{A_p}}$(x) is given below:

$$\mu_{\widetilde{A_p}}(x) = \begin{cases} 0 & x \le a_1 \\ \frac{w}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ \frac{w}{2}+\frac{w}{2}\left(\frac{x-a_2}{a_3-a_2}\right) & a_2 \le x \le a_3 \\ w & x = a_3 \\ w-\frac{w}{2}\left(\frac{x-a_3}{a_4-a_3}\right) & a_3 \le x \le a_4 \\ \frac{w}{2}\left(\frac{a_5-x}{a_5-a_4}\right) & a_4 \le x \le a_5 \\ 0 & x \ge a_5 \end{cases}$$

4.2.2. The α- cut of a normal heptagonal fuzzy number $\widetilde{A_H} = (a_1, a_2, a_3, a_4, a_5)$ is

$$\tilde{A} = [A_\alpha{}^L, A_\alpha{}^R] = \begin{cases} \left(A_\alpha{}^L\right)_1, \left(A_\alpha{}^R\right)_1 & \alpha \in [0,k] \\ \left(A_\alpha{}^L\right)_2, \left(A_\alpha{}^R\right)_2 & \alpha \in [k,1] \end{cases}$$

Where $\left(A_\alpha{}^L\right)_1 = a_1 + \frac{r}{k}(a_2 - a_1)$, $\left(A_\alpha{}^L\right)_2 = a_2 + \frac{1-s}{1-k}(a_3 - a_2)$,

$\left(A_\alpha{}^R\right)_1 = a_{4-} \frac{1-s}{1-k}(a_3 - a_2)$, $\left(A_\alpha{}^R\right)_2 = a_5 - \frac{r}{k}(a_5 - a_4)$

4.3. Proposed Ranking Technique:

$$R(\tilde{A}_H) = \frac{1}{2}\int_0^k \{l_1(r) + l_2(r)\}dr + \frac{1}{2}\int_k^1 \{s_1(t) + s_2(t)\}ds$$

$$R(\tilde{A}_H) = \frac{1}{2}\int_0^k [a_1 + \frac{r}{k}(a_2 - a_1) + a_{5-} \frac{r}{k}(a_5 - a_4)]dr + \frac{1}{2}\int_k^1 [a_2 + \frac{1-s}{1-k}(a_3 - a_3) +$$

$$a_4 - \frac{1-s}{1-k}(a_4 - a_3)]ds$$

$$R(\tilde{A}_H) = \frac{1}{2}\Big\{(a_1 + a_5)r]_k^0 + \frac{(a_2 - a_1)}{k}\frac{r^2}{2}]_k^0 - \frac{(a_5 - a_4)}{k}\frac{r^2}{2}]_k^0 + (a_2 + a_4)s]_k^0$$

$$+ \frac{(a_3 - a_2)}{1-k}s]_k^1 + \frac{(a_3 - a_2)}{1-k}\frac{s^2}{2}]_k^1 - \frac{(a_4 - a_3)}{1-k}s]_k^1 + \frac{(a_4 - a_3)}{1-k}\frac{s^2}{2}]_k^1\Big\}$$

$$= \frac{1}{2}\{(a_1 + a_5)k + \frac{(a_2-a_1)}{k}\frac{k^2}{2} - \frac{(a_5-a_4)}{k}\frac{k^2}{2} + (a_2 + a_4) - (a_2 + a_4)k + \frac{(a_3-a_2)}{1-k} -$$

$$\frac{(a_3-a_2)}{1-k}k - \frac{(a_3-a_2)}{1-k}\frac{1}{2} + \frac{(a_3-a_2)}{1-k}\frac{k^2}{2} - \frac{(a_4-a_3)}{1-k} + \frac{(a_4-a_3)}{1-k}k + \frac{(a_4-a_3)}{1-k}\frac{1}{2} - \frac{(a_4-a_3)}{1-k}\frac{k^2}{2}\}$$

$$= \frac{1}{4}\{(a_1 + 2a_3 + 2a_4 + a_5)k + (a_2 - 2a_3 + a_4)\}$$

4.4Application of ranking of fuzzy numbers to Game theory:

4.4.1 Consider the following fuzzy game problem with payoff as pentagonal fuzzy numbers

B

$$A\begin{bmatrix} (7,8,9,10,11)(2,0,2,4,6) & (-2,0,2,3,4) \\ (0,1,2,3,4,)(1,3,5,7,9)(-4,0,2,4,8) \\ (4,6,8,10,12)(4,5,6,7,8) & (3,6,9,12,15) \end{bmatrix}$$

By definition of normal pentagonal fuzzy number $\tilde{A}$ is calculated as defined in section 4.3 convert the fuzzy game problem into crisp problem, then solved by maximin-minimax principle and find the value of the game.

$\tilde{a}_{11} = (7,8,9,10,11)$ $M_0^p(\tilde{a}_{11})=$ 28

$\tilde{a}_{12} = (2,0,2,4,6)$ $M_0^p(\tilde{a}_{12})=$ 10

$\tilde{a}_{13} = (-2,0,2,3,4)$ $M_0^p(\tilde{a}_{13})=$ 5

$\tilde{a}_{21} = (0,1,2,3,4)$ $M_0^p(\tilde{a}_{21})=$ 7

$\tilde{a}_{22} = (1,3,5,7,9)$ $M_0^p(\tilde{a}_{22})=$ 17

$\tilde{a}_{23} = (-4,0,2,4,8)$ $M_0^p(\tilde{a}_{23})=$ 8

$\tilde{a}_{31} = (4,6,8,10,12)$ $M_0^p(\tilde{a}_{31})=$ 26

$\tilde{a}_{32} = (4,5,6,7,8)$ $M_0^p(\tilde{a}_{32})=$ 19

$\tilde{a}_{33} = (3,6,9,12,15)$ $\boldsymbol{M_0^p(\tilde{a}_{33})}=$ 30

The given fuzzy game problem is reduced in the following payoff matrix

B

$$A \begin{bmatrix} 28 & 10 & 5 \\ 7 & 17 & 8 \\ 26 & 19 & 30 \end{bmatrix}$$

Minimum of 1st row = 5

Minimum of 2nd row = 7

Minimum of 3rd row = 19

Maximum of 1st column = 28

Maximum of 2nd column = 19

Maximum of 3rd column = 30

Max (min) = 19 and Min (max) =19

Here Max (min) $=$ Min (max)

It has equilibrium point and the value of Game = 19

4.4.2 Consider the following fuzzy game problem with payoffs as :

B

$$A \begin{bmatrix} (-0.1,0,0.3,0.5,0.7)(-3,0,3,7,10)(-1,0,1,2,3) \\ (0,2,3,4,5)(0,2,4,6,8)(-2,0,,2,3,4) \\ (0.8,0.7,0.5,0.3,0.2) \quad (7,9,12,15,18) \quad (-4,7,9,10,13) \end{bmatrix}$$

By definition of normal pentagonal fuzzy number $\tilde{\boldsymbol{A}}$ is calculated as defined in section 4.3 convert the fuzzy game problem into crisp problem, then solve by oddment method.

Step1. We obtain the values of $\boldsymbol{M_0^p}$ $(\tilde{\boldsymbol{a}}_{\boldsymbol{ij}})$ of the given fuzzy game problem and convert the fuzzy game into crisp value problem which is given in the following table:

$\tilde{a}_{11} = (-0.1,0,0.3,0.5,0.7)$	$M_0^p(\tilde{a}_{11})=$ 0.34
$\tilde{a}_{12} = (-3,0,3,7,10)$	$M_0^p(\tilde{a}_{12})=$ 6.4
$\tilde{a}_{13} = (-1,0,1,2,3)$	$M_0^p(\tilde{a}_{13})=$ 4

$\tilde{a}_{21} = (0,2,3,4,5)$ $M_0^p(\tilde{a}_{21})=$ 9.5

$\tilde{a}_{22} = (0,2,4,6,8)$ $M_0^p(\tilde{a}_{22})=$ 14

$\tilde{a}_{23} = (-2,0,2,3,4)$ $M_0^p(\tilde{a}_{23})=$ 5

$\tilde{a}_{31} = (0.8,0.7,0.5,0.3,0.2)$ $M_0^p(\tilde{a}_{31})=$ 1.3

$\tilde{a}_{32} = (7,9,12,15,18)$ $M_0^p(\tilde{a}_{32})=$ 31.6

$\tilde{a}_{32} = (-4,7,9,10,13)$ $M_0^p(\tilde{a}_{33})=$ 13.1

Step2. The given fuzzy game problem is reduced in the following payoff matrix:

B

$$A \quad \begin{bmatrix} 0.34 & 6.54 & 4 \\ 9.5 & 14 & 5 \\ 1.3 & 31.6 & 13.1 \end{bmatrix}$$

Minimum of 1st row = 0.34

Minimum of 2nd row = 5

Minimum of 3rd row = 1.3

Maximum of 1st column = 9.5

Maximum of 2nd column = 31.6

Maximum of 3rd column = 13.1

Max (min) = 5 and Min (max) = 9.5

Here Max (min) $\neq$ Min (max)

It has no saddle point.

Step3. To solve the reduced crisp value problem, we apply dominance method. Clearly second column is dominated by first column as all the elements of second column are greater than first column. Hence eliminating second column, we get

B

A $\begin{bmatrix} 0.34 & 4 \\ 9.5 & 5 \\ 1.3 & 13.1 \end{bmatrix}$

Again, first row is dominated by the second row as all the elements of first row are less than second row. Hence eliminating first row, we get

B

A $\begin{bmatrix} 9.5 & 5 \\ 1.3 & 13.1 \end{bmatrix}$

Now we obtain 2×2 payoff matrix, Since the reduced matrix do not have any saddle point, so we apply oddment method. Thus the augmented payoff matrix is

			Row oddments
	9.5	5	11.8
	1.3	13.1	4.5
Column oddments	8.1	8.2	16.3

Since the sum of row oddments and column oddments equal to 16.3, the optimum strategies are:

Row player ($0.723, 0.276$) and column player ($0.4969, 0.5030$)

The value of the game is 7.2268.

Conclusion: The chapter describes ranking method for pentagonal fuzzy numbers to solve fuzzy game problems. This new ranking method is based on $\boldsymbol{\alpha}$ -cut and plays an important role in real life situations. By using this method fuzzy numbers are converted into crisp numbers so that game problem can be solved by traditional method. In the above example 1, the payoffs are pentagonal fuzzy numbers. By using the proposed ranking technique pentagonal fuzzy numbers are converted into crisp and then find the saddle point. In this problem, the players A choses his pure strategy $A_3$to maximize their minimum gaines and player B choses his B_2 to minimize their maximum losses and the value of the game is 19 units. In the second example there does not exist a saddle point, so the player uses their optimum mixed strategies. After converting the payoff matrix into crisp problem, we use dominance property to reduce the payoff matrix into $2\times \mathbf{2}$. Now applying the oddment method and find that Player A will select A_3 and A_3 strategies to maximize his minimum gaines with probabilities 0.33 and 0.67 respectively and player B will select B_2 and B_3 with probabilities 0.83 and 0.17 respectively to minimize his maximum loses. Hence value of the game is 5.83.

Chapter 5

SOLUTION OF FUZZY GAME PROBLEM BY USING DODECAGONAL FUZZY NUMBERS

5.1 INTRODUCTION

The fuzzy set theory has been applied in almost every business enterprise as well as day to day activity. Ranking of fuzzy numbers plays an important role in decision making process. In this paper, we introduced a fuzzy game problem (FGP) in which the values of payoff matrix are represented by dodecagonal fuzzy numbers. By using ranking to payoffs we convert the fuzzy game problem into crisp valued game problem, and then solve it by using by using any method.

The concept of fuzzy set theory deals with imprecision, vagueness in real life situations. It was firstly proposed by Zadeh [1]. Bellman and Zadeh [2] elaborated on the concept of decision making in the fuzzy environment. Later on, fuzzy methodologies have been successfully applied in a wide range of real world situations. Jain [3] was the first to propose method of ranking fuzzy numbers for decision making in fuzzy situations. Yager [4] used the concept of centroids in the ranking of fuzzy numbers.

In Fuzzy Game Problems, all parameters are fuzzy numbers. Fuzzy numbers may be normal or abnormal, triangular, trapezoidal, hexagon, octagon etc. Jatinder and Neha [33] proposed the ranking method for ordering dodecagonal fuzzy numbers based on rank, mode, divergence and spread. Selvakumari and lavanya [27] considered an approach for solving fuzzy game problem in which payoff are triangular and trapezoidal fuzzy numbers. Selvakumari and lavanya [22] considered a ranking of octagonal fuzzy numbers to solve fuzzy game problem.

5.2 Dodecagonal Fuzzy Numbers: Ageneralized fuzzy number $\widetilde{A_D} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}; w)$ is said to be dodecagonal fuzzy number if its membership function $\mu_{\widetilde{A_D}}(x)$ is given below:

$$\boldsymbol{\mu}_{\widetilde{A_D}}(x) = \begin{cases} 0 & x \le a_1 \\ k_1\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ k_1 & a_2 \le x \le a_3 \\ k_1 + (1-k_1)\left(\frac{x-a_3}{a_4-a_3}\right) & a_3 \le x \le a_4 \\ k_2 & a_4 \le x \le a_5 \\ \text{k2} + (1-\text{k2})\left(\frac{a_8-x}{a_8-a_7}\right) & a_5 \le x \le a_6 \\ 1 & a_6 \le x \le a_7 \\ \text{k2} + (1-\text{k2})\left(\frac{x-a_5}{a_6-a_5}\right) & a_7 \le x \le a_8 \\ k_2 & a_8 \le x \le a_9 \\ k_1 + (1-k_1)\left(\frac{a_{10}-x}{a_{10}-a_9}\right) & a_9 \le x \le a_{10} \\ k_1 & a_{10} \le x \le a_{11} \\ k_1\left(\frac{a_2-x}{a_2-a_1}\right) & a_{11} \le x \le a_{12} \\ 0 & x \ge a_{12} \end{cases}$$

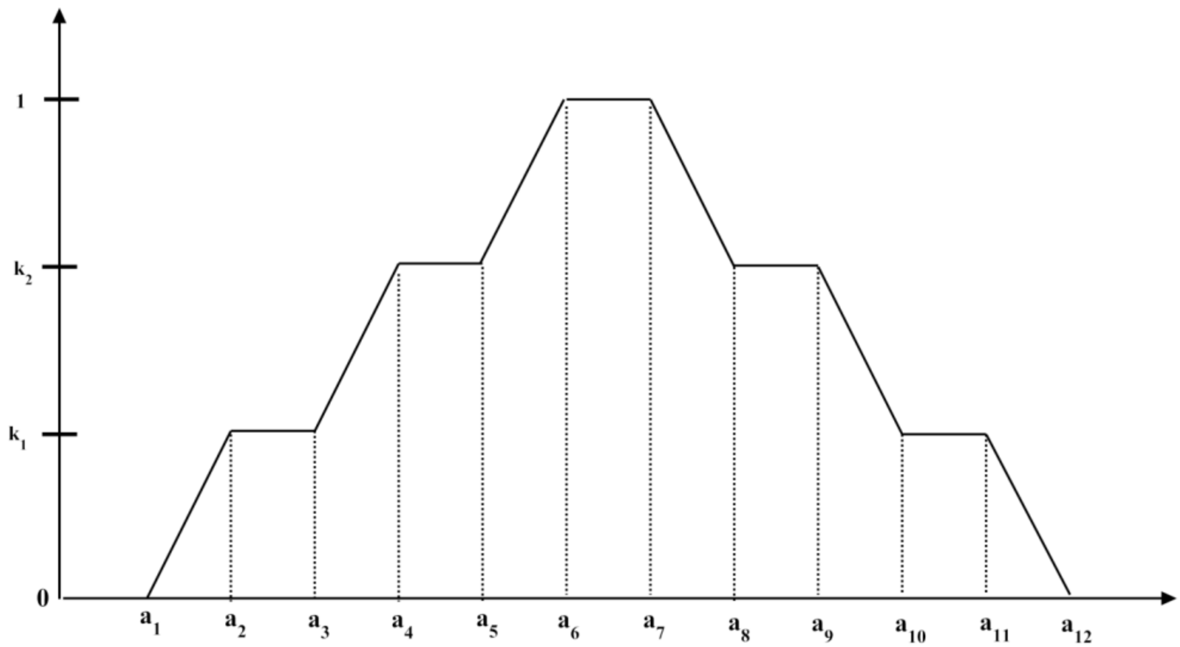


Fig: 5.1

5.3 Ranking of Dodecagonal Fuzzy Numbers: Let $\tilde{A}$ be a normal dodecagonal fuzzy number. The value $M_0^{Dod}(\tilde{A})$, called the measure of $\tilde{A}$ is calculated as follows:

$$M_0^{Dod}(\tilde{A}) = \frac{1}{2}\int_1^{k_1}(f_1(r) + f_2(r))dr + \frac{1}{2}\int_{k_1}^{k_2}(g_1(s) + g_2(s))dr + \frac{1}{2}\int_{k_2}^{1}(h_1(t) + h_2(t))dr$$

$$M_0^{Dod}(\tilde{A})=\frac{1}{4}\{(a_1 + a_2 + a_{11} + a_{12})k_1 + (a_3 + a_4 + a_9 + a_{10})(k_2 - k_1) + (a_5 + a_6 + a_7 + a_8)(1 - k_2)\}$$

Where $0< k_1 < k_2 <1$

5.4 NUMERICAL EXAMPLES:

5.4.1 Consider the following fuzzy game problem with payoff as dodecagonal fuzzy numbers

B

A $\begin{bmatrix} (-3,-2,-1,0,1,2,3,4,5,6,7,8) & (2,3,4,5,6,7,8,9,10,11,12,13) \\ (-5,-4,-3,-2,-1,0,1,2,3,4,5,6) & (0,1,2,3,4,5,6,7,8,9,10,11) \\ (8,9,11,12,14,15,16,17,18,21,22,23) & (2,3,5,6,8,9,10,11,12,15,16,17) \end{bmatrix}$

Solution: By definition of dodecagonal fuzzy number $\tilde{A}$ is calculated as $M_0^{Dod}(\tilde{A}) = \frac{1}{4}\{(a_1 + a_2 + a_{11} + a_{12})k_1 + (a_3 + a_4 + a_9 + a_{10})(k_2 - k_1) + (a_5 + a_6 + a_7 + a_8)(1 - k_2)\}$ Where $0 < k_1 < k_2 < 1$

This problem is done by taking the values of $k_1 = 0.4$ and $k_2 = 0.8$, we obtain the values of $M_0^{Dod}(\widetilde{a_{ij}})$.

$\tilde{a}_{11} = (-3,-2,-1,0,1,2,3,4,5,6,7,8)$ $M_0^{Dod}(\tilde{a}_{11}) =$ 2.5

$\tilde{a}_{12} = (2,3,4,5,6,7,8,9,10,11,12,13)$ $M_0^{Dod}(\tilde{a}_{12}) =$ 7.5

$\tilde{a}_{21} = (-5,-4,-3,-2,-1,0,1,2,3,4,5,6)$ $M_0^{Dod}(\tilde{a}_{21}) =$ 0.5

$\tilde{a}_{22} = (0,1,2,3,4,5,6,7,8,9,10,11)$ $M_0^{Dod}(\tilde{a}_{22}) =$ 5.5

$\tilde{a}_{31} = (8,9,11,12,14,15,16,17,18,21,22,23)$ $M_0^{Dod}(\tilde{a}_{31}) =$ 15.5

$\tilde{a}_{32} = (2,3,5,6,8,9,10,11,12,15,16,17)$ $M_0^{Dod}(\tilde{a}_{32}) =$ 9.5

The given fuzzy game problem is reduced in the following payoff matrix

B

$$A \begin{bmatrix} 2.5 & 7.5 \\ 0.5 & 5.5 \\ 15.5 & 9.5 \end{bmatrix}$$

Minimum of 1st row = 2.5

Minimum of 2nd row = 0.5

Minimum of 3rd row = 9.5

Maximum of 1st column = 15.5

Maximum of 2nd column = 9.5

Max(min) = 9.5 and Min(max) =9.5

It has saddle point. Value of Game = 9.5

5.4.2 Consider the following fuzzy game problem :

$$\begin{array}{c} \\ A \end{array} \overset{B}{\begin{bmatrix} (-3,-2,-1,0,1,2,3,4,5,6,7,8) & (6,7,8,9,10,11,12,13,14,15,16,17) & (2,3,4,5,6,7,8,9,10,11,12,13) \\ (-5,-4,-3,-2,-1,0,1,2,3,4,5,6) & (2,3,5,6,8,9,10,11,12,15,16,17) & (-5,-4,-3,-1,0,1,2,4,5,6,7,9) \\ (0,1,2,4,5,6,7,8,9,11,12,13) & (1,2,3,6,7,8,9,10,12,13,15,16) & (-2,-1,0,1,2,3,4,5,6,7,8,9,) \end{bmatrix}}$$

By definition of dodecagonal fuzzy number $\tilde{A}$ is calculated as $M_0^{Dod}(\tilde{A})=\frac{1}{4}\{(a_1 + a_2 + a_{11} + a_{12})k_1 + (a_3 + a_4 + a_9 + a_{10})(k_2 - k_1) + (a_5 + a_6 + a_7 + a_8)(1 - k_2)\}$ Where $0 < k_1 < k_2 < 1$

Step 1. We obtain the values of $M(\tilde{a}_{ij})$ of the given fuzzy game problem and convert the fuzzy game into crisp value problem which is given in the following table:

$\tilde{a}_{11} = (-3,-2,-1,0,1,2,3,4,5,6,7,8)$ $M_0^{Dod}(\tilde{a}_{11})=$ 2.5

$\tilde{a}_{12} = (6,7,8,9,10,11,12,13,14,15,16,17)$ $M_0^{Dod}(\tilde{a}_{12})=$ 11.5

$\tilde{a}_{13} = (2,3,4,5,6,7,8,9,10,11,12,13)$ $M_0^{Dod}(\tilde{a}_{13})=$ 7.5

$\tilde{a}_{21} = (-5,-4,-3,-2,-1,0,1,2,3,4,5,6)$ $M_0^{Dod}(\tilde{a}_{21})=$ 0.5

$\tilde{a}_{22} = (2,3,5,6,8,9,10,11,12,15,16,17)$ $M_0^{Dod}(\tilde{a}_{22})=$ 9.5

$\tilde{a}_{23} = (-5,-4,-3,-1,0,1,2,4,5,6,7,9)$ $M_0^{Dod}(\tilde{a}_{32})=$ 1.75

$\tilde{a}_{31} = (0,1,2,4,5,6,7,8,9,11,12,13)$ $M_0^{Dod}(\tilde{a}_{31})=$ 6.5

$\tilde{a}_{32} = (1,2,3,6,7,8,9,10,12,13,15,16)$ $M_0^{Dod}(\tilde{a}_{32})=$ 8.5

$\tilde{a}_{33} = (-2,-1,0,1,2,3,4,5,6,7,8,9,)$ $M_0^{Dod}(\tilde{a}_{33})=$ 3.5

Step 2. The given fuzzy game problem is reduced in the following payoff matrix

B

$$A \quad \begin{bmatrix} 2.5 & 11.5 & 7.5 \\ 0.5 & 9.5 & 1.75 \\ 6.5 & 8.5 & 3.5 \end{bmatrix}$$

Minimum of 1st row = 2.5

Minimum of 2nd row = 0.5

Minimum of 3rd row = 3.5

Maximum of 1st column = 6.5

Maximum of 2^{nd}column = 11.5

Maximum of 3^{rd}column = 7.5

Max(min) = 3.5 and Min(max) =6.5

Here Max(min) $\neq$ Min(max)

It has no saddle point.

Step3. To solve the reduced crisp value problem,we apply dominance method.Clearly second row is dominated by first row as all the elements of second row are less than first row. Hence eliminating second row,we get

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 2.5 & 11.5 & 7.5 \\ 6.5 & 8.5 & 3.5 \end{bmatrix} \end{array}$$

Again, Second column is dominated by the first column as all the elements of second column are greater than first column. Hence eliminating second column, we get

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 2.5 & 7.5 \\ 6.5 & 3.5 \end{bmatrix} \end{array}$$

Now we obtain 2×2 paypff matrix , Since the reduced matrix donnot have any saddle point , so we apply oddment method .Thus the augmented payoff matrix is

Row oddments

	2.5	7.5	3
	6.5	3.5	5
Column oddments	4	4	8

Since the sum of row oddments and column oddments equal to 8, the optimum strategies are:

Row player $(\frac{3}{8}, \frac{5}{8})$ and column player $(\frac{1}{2}, \frac{1}{2})$

The value of the game is 3.

The process of ranking is used to convert the fuzzy game problem into crisp value problem and applied dominance property and then solved by oddment method to find the value of the game and the optimum strategies.In this chapter ranking of dodecagonal fuzzy number is used to convert fuzzy game problem into crisp problem and then solve it by using traditional method.

Result and Discussion

In this Project the fuzzy game theory for decision making process is analyzed. In this different fuzzy numbers are used to solve fuzzy game problems. In the fuzzy game problems all the payoffs are fuzzy. The fuzzy numbers are Triangular, Trapezoidal, Pentagonal fuzzy numbers, heptagonal fuzzy numbers, Octagonal fuzzy numbers, Dodecagonal fuzzy numbers etc. These fuzzy numbers are converted into crisp numbers by using different ranking techniques and then solve fuzzy game problems by using any traditional method. In the first chapter, Introduction and Literature review is discussed. The traditional game theory assumes the existence of exact payoffs to solve competitive situations. However in the real life game situations such precise information on the payoffs is not available. Due to lack of information, the players are not able to estimate exactly payoffs in real situations. This lack of certainty may be appropriately modeled by using fuzzy set.In fuzzy environment ranking of fuzzy numbers is very important aspect of decision making. Many authors have proposed different methods for ranking of fuzzy numbers.

In the Second chapter, the definitions of fuzzy number, fuzzy set, Triangular fuzzy number, Trapezoidal fuzzy number, Pentagonal fuzzy number, Octagonal fuzzy number etc. are given. In this chapter, Fuzzy game problem and its mathematical formulation is also discussed.

In the third chapter, the payoffs of fuzzy game problem are octagonal fuzzy numbers. This chapter describes the method of ranking octagonal fuzzy numbers using centroid of centroids and incentre of centroids. In octagonal fuzzy number, firstly the octagon is split into two trapezoidal and one hexagon and then computes the centroid of these plane figures. Secondly, the centroid of these centroids is followed by calculation of incentre. In this chapter, the method of ranking fuzzy numbers with an area between the incentre and the original point is also introduced.

The fourth chapter describes ranking method for pentagonal fuzzy numbers to solve fuzzy game problems. This new ranking method is based on $\boldsymbol{\alpha}$ -cut and plays an important role in real life situations. By using this method fuzzy numbers are converted into crisp numbers so that game problem can be solved by traditional method.

In fifth chapter a method of solving fuzzy game problem by using ranking of dodecagonal fuzzy numbers has been considered. By using the proposed ranking method, a crisp game problem is obtained and finds the optimum mixed strategies by using the dominance property. It is concluded that in comparison to classical game techniques, using of fuzzy game techniques under unclearness, and incompleteness of information is more preferable.

References:

[1] Nash, J.S (1949),Equilibrum points in n-Person games,*Princeton University Press*.

[2] Zadeh, L.A. (1965), Fuzzy sets, *Information and Control,***8**(3), 338-353.

[3] Bellman R.E., Zadeh L.A. (1970), Decision making in fuzzy environment, *Management Science*, **17**(4), 141- 164.

[4] Jain, R. (1976), Decision making in the presence of fuzzy variables, *IEEE Transactions on Systems, Man and Cybernetics*, 6, 698-703.

[5] Yager, R. R. (1980), On a general class of fuzzy connectives, *Fuzzy Sets and Systems,***4**(6), 235-242.

[6] M. Sakawa, I. Nishizaki, (1994), Max–min solutions for fuzzy multiobjective matrix games, *Fuzzy Sets and Systems*,**67**(1), 53–69.

[7] Chu, T.C. and Tsao, C.T. (2002), Ranking fuzzy numbers with an area between the centroid point, *Computers and Mathematics with Applications,***43**, 111-117.

[8]Bector C.R., Chandra S. (2005), Fuzzy Mathematical Programming and Fuzzy Matrix Games, Studies in Fuzziness and Soft Computing, **69.**

[9] Vijay V, Mehra A, Chandra S, Bector C.R.,(2007), Fuzzy matrix games via a fuzzy relation approach, *Fuzzy Optimization and Decision Making*, **6** , 299–31

[10] Cevikel, A.C and Ahlatcioglu, M. (2010), Solution for fuzzy matrix games, *Computers and Mathematics with Applications*,**60**, 399 - 410.

[11] Kumar A, Singh P,Kaur P, Kaur A (2010), Equality of generalized Triangular Fuzzy Numbers, *International Journal of Physical and Mathematical Sciences*,**1**,43-48.

[12] Çevikel A.Cand Ahlatçıoglu M (2010), A linear interactive solution concept for multiobjective Games , *European Journal of Pure and Applied Mathematics* ,**3**(1), 107-117.

[13] Nayak P.K,Pal M (2012),Matrix Games with Intuitionistic Fuzzy Goals, *International Journal of Engineering Research and Development* ,**4**(1),01-10.

[14] Thorani, Y.L.P, Rao, P.P.B and Shankar, N.R (2012) ,Ordering generalized trapezoidal fuzzy number using orthocentre of centroids, *International Journal of Algebra,* **6** *(22)*, 1069-1085.

[15] R.Parvathi ,C.Malathi(2012),Arithmetic Operations on Symmetric Trapezoidal Intuitionistic Fuzzy Numbers, *International Journal of Soft Computing and Engineering*, **2**(2), 268.

[16] Seikh M R, Pal M, Nayak P.K (2012), Application of Triangular Intuitionistic Fuzzy Numbers in Bi-Matrix Games, *Intenational Journal of Pure and Applied Mathematics*,**79**(2), 235-247.

[17] Babu, S.S., Thorani, Y.L.P., Shankar, N.R.(2012), Ranking generalised fuzzy numbers using centroid of centroids, *International Journal of Fuzzy Logic Systems (IJFLS)*, **2** (3), 17-32

[18] P. Rajarajeswari , A SahayaSudha and R. Karthika.(2013), A New OperationOn Hexagonal Fuzzy Number,International Journal of Fuzzy Logic Systems, **3**(3),15-26.

[19] Kumar S, Chopra R and Saxena R.R (2013). Method to solve fuzzy game matrix, International journal of pure and applied mathematics, **89**(5), 279-287.

[20]Rajarajeswari, P and Sudha, A.S (2014), A new Approach for Ranking of fuzzy number using Incentre of centroids, *International Journal of Fuzzy Mathematical Archive,***4**(1), 52-60.

[21] Lee B and Yun Y.S (2014), The Pentagonal fuzzy numbers, *Journal of the Chungcheong Mathematical society*,**27**(2), 277-286.

[22] SelvakumariKand Lavanya S (2014),On Solving Fuzzy Game Problem using Octagonal Fuzzy Numbers, *Annals of Pure and Applied Mathematics* ,**8**(2), 211-217

[23] Dhanalakshmi, V. and Kennedy, F.C. (2014), Some ranking methods for octagonal fuzzy numbers, *International Journal of Mathematical Archive,***5**(6), 177-188.

[24]Kamble, A.J and T.Venkatesh (2014),Some results on fuzzy numbers, Annals of Pure and Applied Mathematics, **7**(2), 90-97

[25]Rajarajeswari, P and Sudha,A.S (2014), Ordering generalized hexagonal fuzzy numbers using rank, mode, divergence and spread, *IOSR Journal of Mathematics (IOSR-JM),* **10**(3),15-22.

[26]*ThangarajBeaula*and *Priyadharsini M (2015)*, Operations on Intuitionistic Trapezoidal Fuzzy Numbers using Interval Arithmetic,*International Journal Fuzzy Mathematical Archive*,**9**(1),125-133.

[27] Selvakumari, K and Lavanya, S (2015), An Approach for Solving Fuzzy Game Problem,'Indian Journal of Science and Technology, **8**(15).

[28]Seikh M.R, Nayak P.K , Pal M, (2015), An alternative Approach for Solving Fuzzy Matrix Games, *International Journal of Mathematics and Soft Computing*,**5**(1),79-92.

[29]MajidMousavi and SalimRezvani (2015), Rank, Mode, Divergence and Spread on Generalized Triangular Fuzzy Numbers, *MathematicaAeterna*, **5**(3), 403 – 415.

[30] R. Senthil Kumar, S. Kumaraghuru (2015), Solution of Fuzzy Game Problem Using Triangular Fuzzy Number,*International Journal of Innovative Science, Engineering & Technology*, **2**(2).

[31] SharmaS.C andGanesh Kumar, (2015), An Algorithm to Solve the Games under Incomplete Information, *Annals of Pure and Applied Mathematics*,**10**(2), 221-228

[32] Namarta and Thakur, N.I (2016), Ranking of Hendecagonal Fuzzy Numbers Using Centroid of Centroids. *Aryabhatta Journal of Mathematics and Informatics*, **8**(1),127-133.

[33] Singh,J.P and Thakur, N.I.(2016), Ranking of Generalised Dodecagonal Fuzzy Numbers Using Incentre of Centroids, *Journal of Mathematics and Informatics*, **5**, 11-15.

[34] Lotfi M, Salahshor S, Esfahani F.N, and Jafarnejad A. (2016), A new approach for ranking fuzzy numbers, *Journal of fuzzy set value analysis*,(1), 102-109.

[35] Savitha M.T and George M. (2017), New methods for ranking of trapezoidal fuzzy numbers, *Advances in fuzzy mathematics*, **12**(5), 1159-1170.

[36] Monisha P. and Sangeetha K. (2017), To solve fuzzy game problem ysing pentagonal fuzzy numbers, *International journal for modern trends in Science and Technology*, **3**(9), 152-154.

[37] Thirucheran, M., Kumari, E.R.M and Lavanya, S (2017), A new approach for Solving Fuzzy Game problem,*International Journal of Pure and Applied Mathematics*, **114**(6), 67-75.

[38] Kamble A.J. (2017), Some Notes on Pentagonal Fuzzy Numbers, International Journal of Fuzzy Mathematical Archive, **13**(2), 113-121

[39] Jana J and Roy S.K. (2018). Solution of matrix games with generalized trapezodial fuzzy payoffs, *Fuzzy information and Engineering*, **10**(1), 213-224

[40] Soni M, Mene S and Singh M.M. (2018). Fuzzy mathematical approach to game theory.*International journal of statistics and Applied mathematics*, **3**(1), 275-277.

[41] Krishnaveni G and Kandasamy G . (2018), A new approach for the solution of fuzzy games,*Journal of Physics : Conf. series 1000.*