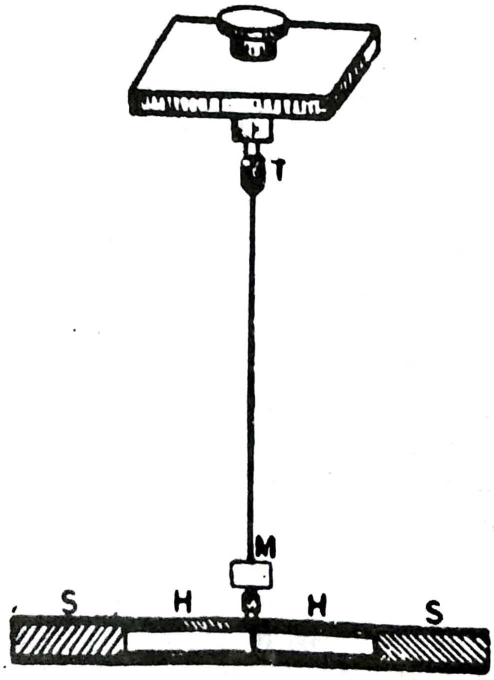

Fig. 5.8

5.10. Maxwell's needle. The modulus of rigidity can be determined with the help of a Maxwell's needle. It consists of a hollow cylindrical tube open at both ends provided with a torsion head in the middle to which is fixed a plane mirror. One end of a wire of uniform radius, modulus of rigidity of the material of which is to be determined is fixed in the torsion head. The other end of the wire is fixed to a second torsion head fixed to a rigid support. The tube is fitted with four cylinders of equal lengths of which two are hollow and two are solid. The length of each cylinder is equal to one quarter of the length of the cylindrical tube so that when they are put end to end they just fill the hollow tube completely. The hollow cylindrical tube along with the cylinders is suspended in a vertical plane as shown in Fig. 5.8. The time period can be determined with the help of a telescope and scale arrangement.

Experiment 9. To determine the modulus of rigidity of copper by Maxwell's needle.

Apparatus. A Maxwell's needle, a copper wire of a suitable length and thickness, a fixed support with a torsion head, a telescope with a scale attached to its stand, a stop-watch, a screw gauge, a spring balance, a metre rod and an electric lamp with a holder.

Theory. To find the co-efficient of rigidity, the Maxwell's needle is suspended first with solid cylinders in the outermost position as shown in Fig. 5.9 (*i*). Let t_1 be the time period. If I_1 is the moment of inertia of the loaded tube about the suspended wire as an axis, then

$$t_1 = 2\pi\sqrt{\frac{I_1}{c}} \quad \text{... } (i)$$

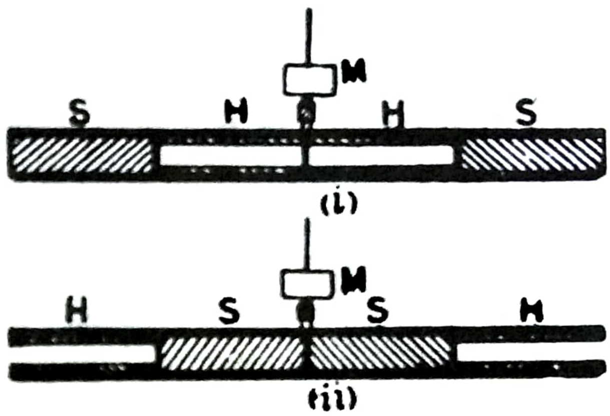

Fig. 5.9

The position of the solid and the hollow cylinders is then changed as shown in Fig. 5.9 (*ii*). Let the time period now be t_2. If I_2 is the moment of inertia of the loaded tube in this position about the wire as an axis, then

$$t_2 = 2\pi\sqrt{\frac{I_2}{c}} \quad \text{... } (ii)$$

From (*i*) and (*ii*), we get

$$t_1^2 - t_2^2 = 4\pi^2\frac{(I_1 - I_2)}{c} \quad \text{... } (iii)$$

Now $$I_1 - I_2 = (m_2 - m_1)L^2 \quad \text{... (iv)}$$

where L is **half** the length of the Maxwell's needle

$$\therefore \quad t_1^2 - t_2^2 = \frac{4\pi^2}{c}(m_2 - m_1)L^2$$

$$= \frac{4\pi^2(m_2 - m_1)L^2 \cdot 2l}{\pi\eta r^4}$$

or $$\eta = \frac{8\pi l(m_2 - m_1)L^2}{(t_1^2 - t_2^2)r^4}$$

Procedure. 1 (*i*) Put the hollow cylinders of the Maxwell's needle inwards and the solid cylinders outwards symmetrically in the tube as shown in Fig. 59 (*i*) (*ii*) Take a copper wire of suitable length and thickness. Remove the kinks by stretching the wire gently.

(*iii*) Suspend the Maxwell's needle from one end of this wire and fix the other end in a rigid support. See that the needle is horizontal and oscillates freely in a horizontal plane about the wire as an axis.

(*iv*) See that the needle lies parallel to the wall. If it is not, adjust the orsion head at the top.

(*v*) Arrest the motion of the needle just near its mean position by a stand placed in contact with it or tie one end of a thread to the needle and its other end to a fixed point.

2. Place a telescope having an inverted centimetre scale clamped to its stand on a stool, at a suitable distance not less than one metre from the Maxwell's needle, with the scale towards the mirror fixed to the needle.

Place your eye just above the telescope and adjust the position of the scale till its image is visible in the mirror. If the image is not clearly visible illuminate the scale by an electric lamp placed in front of the scale.

3. Place the telescope in the position of the eye. Adjust the eye-piece so that the cross-wires are clearly visible. Focus the telescope on the image seen in the mirror and not on the mirror. *The image lies as far behind the mirror as the scale is in front of it.* Observe the reading of the scale in the centre of the image. Adjust the vertical cross-wire so that it coincides with a cm division mark.

4. Gently remove the stand or apply a burning match stick to the thread, as the case may be. The needle will begin to vibrate in a horizontal plane. *See that the amplitude is small and the needle does not oscillate up and down.* If it does so stop it and release it again gently or touch the wire just above the needle with the fingers. *Protect the apparatus from air currents.*

5. When the image of the reference line on the scale just passes the vertical cross-wire, start the stop watch and count **zero**. Count **'one'** when the image of the same mark passes in the same direction and so on.

Find the time for 15 vibrations. Repeat three times.

6. Now place the solid cylinders inwards and the hollow cylinders outwards as shown in Fig. 5.9 (*ii*). Set the telescope again and repeat the observations.

7. Find the weight of each solid cylinder with a spring balance and find the mean. Similarly find the mean weight of a hollow cylinder. Measure the length of the Maxwell's needle and that of the wire from the point of suspension to the point where it is attached to the needle.

8. Measure the diameter of the wire accurately at different points with a screw gauge. At each point measure the diameter in two mutually perpendicular directions.

Note that the radius 'r' occurs as r^4 in the formula and hence a small error in the measurement of diameter will be magnified four times.

Observations.

Position of solid cylinders	Time for 15 vibrations				Time period
	1	2	3	Mean	
Outside					$t_1 =$
Inside					$t_2 =$

Weight of the hollow cylinders = (*i*) (*ii*)

Mean m_1 = kg

Weight of the solid cylinders = (*i*) (*ii*)

Mean m_2 = kg

Length of the needle $2L$ = cm = m

Half the length of the needle L = cm = m

Length of the wire l = cm = m

Diameter of the wire

Least count of the screw gauge = mm =

Zero correction = ± mm

Position	1	2	3	4	5	6	Mean
a							
b							

Mean corrected diameter = mm

Mean corrected radius = mm = m

Calculations. $$\eta = \frac{8\pi l (m_2 - m_1) L^2}{(t_1^2 - t_2^2) r^4} = \quad \text{N/m}^2$$

Actual value from tables = N/m^2

Percentage error =

Precautions. 1. There should be no kinks in the wire.

2. The Maxwell needle should vibrate equally on both sides of the reference mark.

3. The thickness of the wire should be such that it remains taut under the load of the Maxwell needle.

4. The Maxwell needle should remain horizontal and should not vibrate up and down.

5. The amplitude of vibrations should be small so that the wire is not twisted beyond the elastic limit.

6. The telescope should be focused on the image of scale which lies as far behind the mirror as the object is in front of it and not on the mirror.

Exercise 1. *Find the moment of inertia of a cylinder by finding its time period. Co-efficient of rigidity is given.*

Hints : $t = 2\pi\sqrt{\frac{I}{c}}$, where $c = \frac{\pi\eta r^4}{2l}$.

Exercise 2. *Find the torsional couple per unit angular twist of a wire.*

Hints. Suspend the Maxwell's needle from the wire and find the time periods t_1 and t_2 with solid cylinders outside and then inside respectively.

Now
$$t_1^2 - t_2^2 = \frac{4\pi^2L^2(m_2 - m_1)}{c}$$

or
$$c = \frac{4\pi^2L^2(m_2 - m_1)}{t_1^2 - t_2^2}$$

Note. L is half the length of Maxwell's needle.