PART I

PROPERTIES OF MATTER

3

Moment of Inertia

3.1. Moment of inertia *of a body about an axis is defined as the sum of the product of the mass and the square of the distance of different particles of the body from the axis of rotation.*

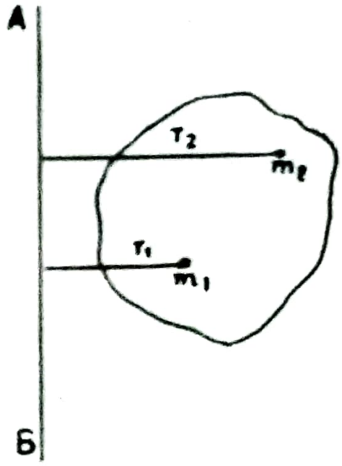

Fig. 3.1

If a body of mass M is supposed to be made up of a large number of small masses m_1, m_2, ... at distances r_1, r_2, ... from the axis of rotation AB, then

The moment of inertia

$$I = m_1 r_1^2 + m_2 r_2^2 + \text{.........}$$
$$= \Sigma m r^2.$$

The $S.I.$ unit of moment of inertia is $kg - m^2$

3.2. Radius of gyration *of a body is the square root of the mean square distance of the particles of the body from the axis of rotation.* If the body is divided into n particles each of mass m and they lie at distances r_1, r_2, ... from the axis of rotation AB, then

$$\text{Radius of gyration } K = \left(\frac{r_1^2 + r_2^2 + \text{.......}}{n}\right)^{\frac{1}{2}}$$

It may also be defined as the distance of the point at which the whole mass of the body may be concentrated so as to have the same moment of inertia.

$$\therefore \qquad I = MK^2.$$

The moment of inertia plays the same role in rotational motion as mass does in linear motion. The $S.I.$ unit of radius of gyration is a metre.

3.3. Energy of rotation of a body. If a body rotates about an axis AB with an angular velocity ω, all its particles have the same angular velocity but different linear velocities as they lie at different distances from the axis of rotation. Let the linear velocities of the particles of mass m_1, m_2, ...from the axis of rotation be v_1, v_2, ... respectively, then

Total kinetic energy of the body = Sum of the kinetic energy possessed by the various particles

$$= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \dots\dots\dots$$

$$= \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + \dots\dots\dots$$

$$= \frac{1}{2}\left(\sum mr^2\right)\omega^2$$

$$= \frac{1}{2}I\,\omega^2$$

where I is the moment of inertia of the body about the axis AB.

3.4. Values of moment of inertia. Moment of inertia of some regular solids is given below :

(*i*) **Circular disc.** The moment of inertia of a circular disc about an axis passing through its $C.G.$ and perpendicular to its plane is given by

$$I = \frac{1}{2}Mr^2$$

where M is the mass of the disc and r its radius.

(*ii*) **A rectangular bar.** The moment of inertia of a rectangular bar about an axis passing through its $C.G.$ and perpendicular to the edges of length l and breadth b is given by

$$I = M\frac{l^2 + b^2}{12}$$

(*iii*) **A right circular cylinder.** The moment of inertia of a right circular cylinder of length l and radius r about an axis through its $C.G.$ and perpendicular to the axis of the cylinder is given by

$$I = M\left(\frac{l^2}{12} + \frac{r^2}{4}\right)$$

(*iv*) **A sphere.** The moment of inertia of a sphere of radius r about a diameter is given by

$$I = \frac{2}{5}Mr^2$$

3.5. Moment of inertia of a flywheel. A flywheel is simply a heavy wheel with a long axle supported in bearings such that it can rest in any position. In other words, the $C.G.$ lies on the axis of rotation.

To find the moment of inertia of a flywheel a mass m is attached to the axle of the wheel by a cord which is wrapped several times round the axle as shown in Fig. 3.2. One end of the string is in the form of a loop so that it can easily be attached to or detached from a pin A projecting from the axle. The length of the string is so adjusted that it gets detached from the axle as soon as the bottom of the mass m is just to touch the floor.

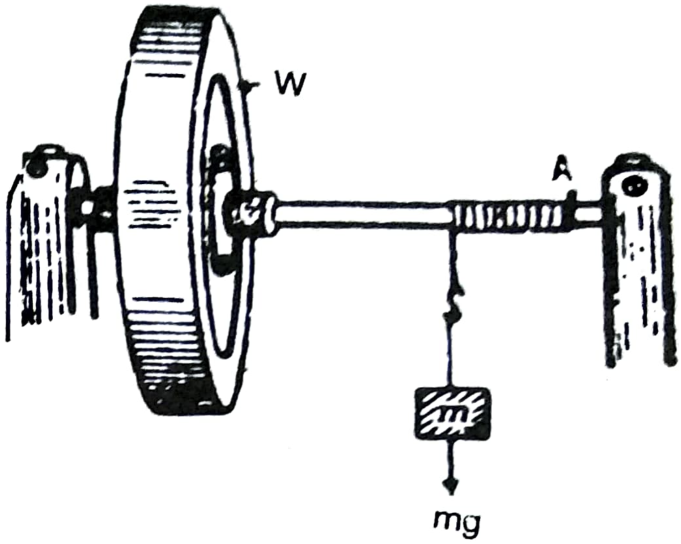

Fig. 3.2

When the mass is allowed to fall, its potential energy is partly converted into the kinetic energy due to the velocity gained by it and partly into the energy of rotation of the flywheel.

Let ω be the angular velocity imparted to the wheel at the moment the mass m is detached.

After the string has been detached from the wheel, the wheel continues to revolve for some time. Its angular velocity decreases on account of friction and finally the wheel comes to rest. If n_1 is the number of revolutions that the wheel makes in time t before coming to rest, then

$$\text{Average angular velocity} = \frac{2\pi n_1}{t}$$

If the frictional force is constant the rotation of the wheel is uniformly retarded. It begins with an angular velocity ω and its final angular velocity is zero, so that the initial velocity ω is double the average velocity.

or $$\omega = 2 \times \frac{2\pi n_1}{t} = \frac{4\pi n_1}{t}$$

According to the principle of conservation of energy, when the string is detached

$$\text{P.E. of mass } m = \text{K.E. of mass } m + \text{K.E. of wheel} + \text{Work done against friction.}$$

If h is the height through which the mass has fallen, then

P.E. of mass $m = mgh$

If r is the radius of the axle, then

Linear velocity of mass $m = r\omega$

and K.E. of mass $m = \frac{1}{2} m (r\omega)^2$

If I is the moment of inertia of the wheel, then

K.E. of the wheel $= \frac{1}{2} I\omega^2$

Let F be the energy per revolution used in overcoming the frictional force. If n is the number of revolutions the wheel makes during the descent of the mass m, then

Total energy used to overcome friction $= nF$

Hence $$mgh = \frac{1}{2} mr^2\omega^2 + \frac{1}{2} I\omega^2 + nF \qquad \text{... }(i)$$

The kinetic energy possessed by the wheel is used up in overcoming friction. As the wheel comes to rest after making n_1 revolutions

$$\therefore \qquad n_1 F = \frac{1}{2} I\omega^2$$

or $$F = \frac{1}{n_1}\frac{1}{2} I\,\omega^2$$

Substituting the value of F in (i), we have

$$mgh = \frac{1}{2} mr^2\omega^2 + \frac{1}{2} I\omega^2 \left(1 + \frac{n}{n_1}\right)$$

or $$I = \frac{2mgh - mr^2\omega^2}{\omega^2\left(1 + {}^{n}\!/_{n_1}\right)}$$

Experiment 1. To find the moment of inertia of a fly-wheel.

Apparatus. A fly wheel, a few different masses and a mass provided with a hook, a strong and thin string, stop watch, a metre rod, a vernier callipers and a piece of chalk.

Formula. Moment of inertia

$$I = \frac{2mgh - mr^2\omega^2}{\omega^2\left(1 + {}^{n}\!/_{n_1}\right)}$$

Procedure. 1. Examine the wheel and see that there is the least possible friction. Oil the bearings, if necessary.

2. Measure the diameter of the axle with a vernier callipers at different points and find the mean. Measure also the circumference of the wheel W with a thread.

3. Take a strong and thin string whose length is less than the height of the axle from the floor. Make a loop at its one end and *slip* it on the pin A on the axle. Tie a suitable mass to the other end of the string. Suspend the mass by means of the string so that the loop is just on the point of slipping from the pin A. Make a chalk mark on the wheel behind the pointer in this position. Also note the position of the lower surface of the mass m on a scale fixed behind on the wall as at C.

4. Now rotate the wheel and wrap the string uniformly round the axle so that the mass m is slightly below the rim of the wheel and the chalk mark is again opposite to the pointer P. Again note the position of the lower surface of the mass on the scale as at B. If now the mass is allowed to fall, it will descend through a height $BC = h$ before being detached from the pin A. Count the number of turns wound round the axle and let it be n. The wheel will thus make n revolutions before the thread is detached.

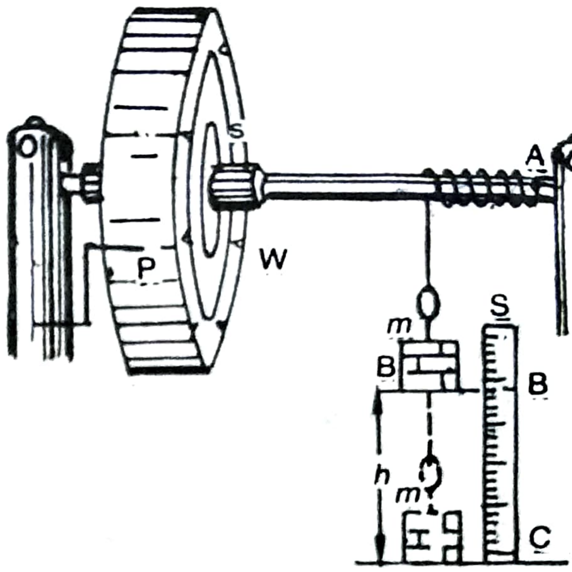

Fig. 3.3

5. Hold a stop-watch in your hand and allow the mass to descend. As soon as the sound of the weight striking the ground is heard, start the stop-watch. Count the number of revolutions n_1 made by the wheel before coming to rest with reference to the chalk mark and note the time t taken for the purpose.

It is possible that in addition to the complete rotations made by the wheel, there may be a fraction of the rotation. To estimate the fraction measure the distance along the circumference by which the chalk mark has advanced beyond the pointer P, by means of a thread. Divide this distance by the circumference of the wheel W. Repeat three times for the same height and load.

6. Repeat the experiment with three different masses, suitably adjusting the height through which the mass falls so that the number of rotations made by the wheel can be easily counted.

Observations. Vernier constant =

	Diameter of the axle		=	(*i*)	(*ii*)	(*iii*)
	Mean diameter of the axle		=		m	
∴	Radius of the axle	r	=		m	
Circumference of the flywheel		W	=		m	
Mass		m	=		kg	
Height		h	=		m	
Number of turns wound on axle		n	=			

Sl. No.	No. of complete revolutions made by the wheel x	Distance of chalk mark from pointer	Fraction of revolution y	No. of revolutions $n_1 = x + y$	Time
1.					
2.					
3.					

Mean $n_1 =$

Mean time $t =$

$\therefore$ Angular velocity $\omega = \dfrac{4\pi n_1}{t}$

Moment of inertia of the flywheel $I = \dfrac{2mgh - mr^2\omega^2}{\omega^2\left(1 + \dfrac{n}{n_1}\right)}$ kg – m^2

Record observations (2) and (3) as above

Mean moment of inertia $I =$ kg – m^2

Precautions. 1. There should be least possible friction in the flywheel. See that the flywheel starts of its own accord and no push is imparted to it. The mass tied to the end of the cord should be of such a value that it is able to overcome friction at the bearings and thus automatically starts falling.

2. The length of the string should be less than the height of the axle of the flywheel from the floor.

3. The loop slipped over the pin should be loose enough to be detached easily.

4. The string should be thin and should be wound evenly.

5. The stop-watch should be started just when the string is detached.

Source of error. (*i*) The angular velocity ω has been calculated on the ssumption that the friction remains constant when the angular velocity lecreases from ω to zero. In actual practice this is not the case because the friction increases as the velocity decreases.

(*ii*) The instant at which the string is detached cannot be correctly found out.